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Model for Unsteadiness in Lateral Dynamics for Use in Parameter Estimation

William R. Wells*
Wright State University, Dayton, Ohio
and
Siva S. Banda† and David L. Quam‡
University of Dayton, Dayton, Ohio

Nomenclature

b = wing span, ft
$\bar{c}$ = mean geometric chord, ft
C_l = rolling moment coefficient, $M_x/\bar{q}S_w b_w$
C_{l_β} = $\partial C_l/\partial\beta$
C_{l_p} = $\partial C_l/\partial(pb/2V)$
C_{l_r} = $\partial C_l/\partial(rb/2V)$
$C_{l_{\delta_a}}$ = $\partial C_l/\partial\delta_a$
C_n = yawing moment coefficient, $M_z/\bar{q}S_w b_w$
C_{n_β} = $\partial C_n/\partial\beta$
C_{n_p} = $\partial C_n/\partial p(b/2V)$
C_{n_r} = $\partial C_n/\partial(rb/2V)$
$C_{n_{\delta_a}}$ = $\partial C_n/\partial\delta_a$
C_y = sideforce coefficient, sideforce $/\bar{q}S_w$
C_{y_β} = $\partial C_y/\partial\beta$
C_{y_p} = $\partial C_y/\partial(pb/2V)$
C_{y_r} = $\partial C_y/\partial(rb/2V)$
g = acceleration due to gravity, ft/s^2
I_x = roll moment of inertia, slug-ft^2
I_z = yaw moment of inertia, slug-ft^2
l_v, Z_v = coordinates of the quarter-chord point of the mean chord of the vertical tail with respect to the center of gravity of the airplane, ft
m = mass, slugs
M_x, M_z = moment about the roll and yaw axes, respectively, lb-ft
p = rate of roll, rad/s
$\bar{q}$ = dynamic pressure, lb/ft^2
r = rate of yaw, rad/s
S = wing surface area, ft^2
t = time, s
V = freestream velocity, ft/s
x = state vector
β = sideslip angle, rad
σ = sidewash angle, rad
ρ = air density, slugs/ft^3
ω = angular frequency, rad/s
ϕ = roll angle, rad
δ_a = aileron deflection, rad
$\Delta\sigma(t)$ = indicial sidewash function
$\Delta C_y(t)$ = indicial sideforce function

Subscripts

F = fuselage
ss = steady state (no unsteady aerodynamic effects)
v = vertical tail
w = wing

Superscript

$(\tilde{\ })$ = Fourier transform of the variable in parentheses

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*Chairman, Dept. of Engineering. Associate Fellow AIAA.
†Graduate Research Associate. Student Member AIAA.
‡Assistant Professor. Member AIAA.

Introduction

During the past two years, some interest and research effort has been given to the role of unsteady aerodynamics in aircraft dynamics. In particular, it was shown that the effects of unsteadiness should be included in a parameter estimation algorithm for longitudinal stability and control derivatives for additional confidence in the estimates. Since existing methods of computing the unsteady lateral derivatives are so limited, this research program was begun.

Analysis

Unsteadiness is introduced into the lateral dynamics via the indicial sideforce and sidewash angle produced by a unit step change in sideslip angle. Due to the changes in magnitude and direction of the flow at the vertical tail from the wing, fuselage and possible propeller slipstream, the effective angle of attack of the vertical tail differs from the sideslip angle by the sidewash angle σ. The sidewash in sideslipping flight is analogous to the downwash in the longitudinal case. Consequently, the change in sidewash angle due to a unit step change in sideslip is assumed to vary as the shed wake moves along the aircraft according to[1]:

$$\Delta\sigma_{l_v}(t) = \left(\frac{\partial\sigma}{\partial\beta}\right)_{ss,l_v}\left\{1 - \frac{F}{\left[\dfrac{l_v\cos(I)}{\bar{c}_w} - I - \dfrac{Vt}{2\bar{c}_w}\right]} - G\exp\left(\frac{-HVt}{\bar{c}_w}\right)\right\} \tag{1}$$

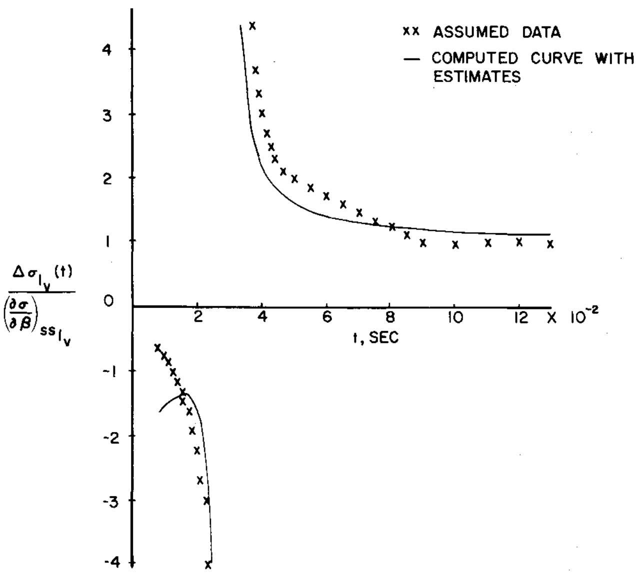

Fig. 1 Comparison of assumed and computed sidewash at the vertical tail.

The constants F, G, and H are to be determined from a curve fit of experimental data or from exact theoretical numerical results.

In order to generate the simulated flight data, it was assumed that the forms of sidewash characteristics are similar to those of downwash. The assumed form of indicial sidewash at the vertical tail is shown in Fig. 1. The singular point is approximately in the region of the projection of the vertical tail length l_v on to the direction of freestream velocity V. Arbitrary values of $\Delta\sigma_{l_v}$ were assumed within a distance of one chord length on both sides of the singular point. Also, it is assumed that the sidewash at the vertical tail would reach its steady-state value after the disturbance has traveled three chord lengths from the singular point. The values at the remaining points were linearly interpolated.

In the calculation of unsteadiness in the side force due to unit step change in the sideslip angle, the wing contribution was neglected and only the vertical tail contribution was considered. Analogous to the indicial lift function in Ref. 4, an approximate expression for the indicial side force was assumed as

$$(\Delta C_y)_v(t) = (C_{y_\beta})_{ss,v}\left[1 - y\exp\left(-z\frac{Vt}{\bar{c}_v/2}\right)\right] \quad (2)$$

The constants y and z are determined from experimental data or from theoretical numerical results.

The sidewash and the vertical tail contribution to the side force for arbitrary changes in sideslip angle were written from Duhamel's integral formula as:

$$\sigma_{l_v}(t) = \int_0^t \Delta\sigma_{l_v}(t-\tau)\dot{\beta}(\tau)\,d\tau \quad (3)$$

$$(C_y)_v(t) = \int_0^t (\Delta C_y)_v(t-\tau)\,[\dot{\beta}(\tau) - \dot{\sigma}_{l_v}(\tau)]\,d\tau \quad (4)$$

The contribution of the vertical tail to the yawing moment and rolling moment are given, respectively, as

$$(C_n)_v(t) = (-S_v l_v / S_w \bar{c}_w)(C_y)_v(t) \quad (5)$$

$$(C_l)_v(t) = (S_v Z_v / S_w \bar{c}_w)(C_y)_v(t) \quad (6)$$

Lateral Equations of Motion

The coupled perturbated equations describing the lateral dynamics of the aircraft are:

$$\dot{\beta}(t) = \frac{q}{V}\phi(t) - r(t) + (\rho V S_w / 2m) C_y(t) \quad (7)$$

$$\dot{p}(t) = (\rho V^2 S_w b_w / 2I_x) C_l(t) \quad (8)$$

Table 1 Aircraft data and flight conditions

Characteristic		Derivative	
m, slugs	91.55	C_{y_r}	5.24
I_x, slug-ft^2	1284.08	C_{y_p}	4.7
I_z, slug-ft^2	3234.72	C_{l_r}	0.11
b_w, ft	33.38	C_{l_p}	−0.49
$\bar{c}_w$, ft	5.7	$C_{l_{\delta_a}}$	0.154
$\bar{c}_v$, ft	3.1	C_{n_r}	−0.09
S_w, ft^2	184	C_{n_p}	−0.04
S_v, ft^2	12.5	$C_{n_{\delta_a}}$	−0.004
l_v, ft	17.13		
Z_v, ft	2		

$V = 240$ ft/s, $\rho = 0.00205$ slugs/ft^3.

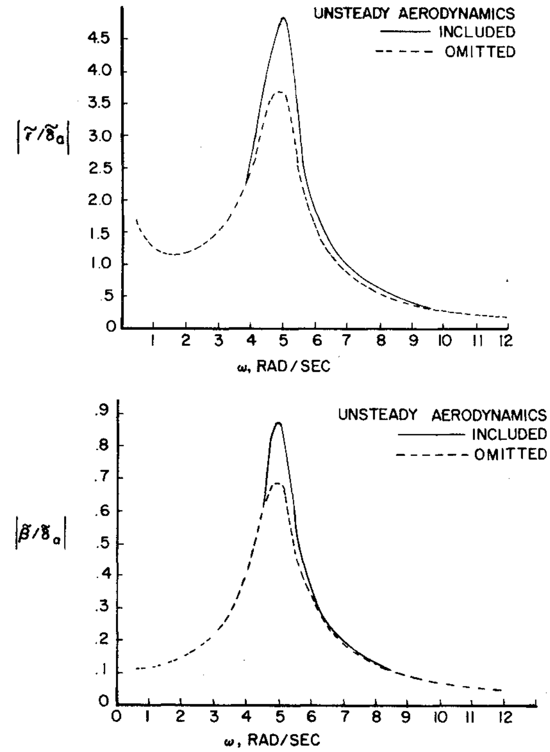

Fig. 2 Comparison of frequency curves with and without unsteady aerodynamics.

$$\dot{r}(t) = (\rho V^2 S_w b_w / 2I_z) C_n(t) \quad (9)$$

$$\dot{\phi} = p(t) \quad (10)$$

The force and moment coefficients expressed in Eqs. (7-10) contain the contributions due to the vertical tail discussed previously and additional effects due to control surface deflections, wing-body combinations, etc. They were taken as:

$$C_y(t) = (S_v/S_w)(C_y)_v(t) + C_{y_r}[r(t)b_w/2V] + C_{y_p}[p(t)b_w/2V] \quad (11)$$

$$C_l(t) = (C_l)_v(t) + (C_{l_\beta})_{w,F}\beta(t) + C_{l_r}[r(t)b_w/2V] + C_{l_p}[p(t)b_w/2V] + Cl_{\delta_a}\delta_a(t) \quad (12)$$

$$C_n(t) = (C_n)_v(t) + (C_{n_\beta})_{w,F}\beta(t) + C_{n_r}[r(t)b_w/2V] + C_{n_p}[p(t)b_w/2V] + C_{n_{\delta_a}}\delta_a(t) \quad (13)$$

Due to computational difficulties in the time domain, the equations of motion were Fourier transformed into the frequency domain. Equations (1) and (2), transformed into the frequency domain, are:

$$jw\cdot\Delta\tilde{\sigma}_{l_v} = \left(\frac{\partial\sigma}{\partial\beta}\right)_{ss,l_v}\left\{1 - \frac{2F\bar{c}_w}{V}\cdot j\omega\cdot\exp\left[-\frac{2(l_v-\bar{c}_w)}{V}j\omega\right]\cdot E_i\left[\frac{2(l_v-\bar{c}_w)}{V}\cdot j\omega\right] - \frac{G\cdot j\omega}{j\omega + (HV/\bar{c}_w)}\right\}$$

$$j\omega\cdot(\Delta\tilde{C}_y)_v = (C_{y_\beta})_{ss,v}\left[1 - \frac{y\cdot j\omega}{[j\omega + (2zV/\bar{c}_v)]}\right] \quad (14)$$

where E_i is an exponential integral.

Numerical Simulation

Computer-simulated frequency response curves were obtained using the data for a lightweight aircraft such as the Navion[5] (see Table 1). Results are presented in Fig. 2 for two cases—one retaining unsteady terms in the equations of motion and the other omitting them (steady-state model). Additional parameters used in obtaining the frequency response curves were obtained as follows:

From Ref. 6:

$$\left(\frac{\partial \sigma}{\partial \beta}\right)_{ss,lv} = -0.1177$$

$$(C_{y_\beta})_{ss,v} = -2.08/\text{rad}$$

$$(C_{l_\beta})_{w,F} = -0.029/\text{rad}$$

and

$$(C_{n_\beta})_{w,F} = -0.229/\text{rad}$$

From Ref. 4:

$$y = 0.151 \text{ and } z = 0.680$$

From Ref. 1:

$$F = 0.318,\ G = 3.04, \text{ and } H = 1.330$$

It was observed that the effect of unsteadiness was present primarily in the sideslip and yaw rate responses, as presented in Fig. 2.

Conclusions

A mathematical model to introduce unsteady aerodynamics into the lateral aircraft dynamics was presented. These effects were illustrated through frequency response curves for such light aircraft as the Navion. The intent was to develop the model in a form suitable for parameter identification in the frequency domain.

Acknowledgment

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Errata

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An Elementary Explanation of the Flutter Mechanism with Active Feedback Controls

H. Horikawa* and E.H. Dowell†
Princeton University, Princeton, N.J.
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IN the above paper three figures were transposed. On page 230, Fig. 8 should be Fig. 9 and Fig. 9 should be Fig. 10. On page 231, Fig. 10 should be Fig. 8.

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*Engineer, Currently with Kawasaki Heavy Industries, Ltd., Kakamigahara, Gifu, Japan. On assignment, The Boeing Company, Seattle, Wash.
†Professor, Department of Mechanical and Aerospace Engineering. Associate Fellow AIAA.